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Studies on the Construction of the Stand Density Control Diagram of Hardwood Forest in France

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Introduction

The process of the growth of plants can be drawn as the logistic curve. The simple logistic curve is expressed as follows.

$$\frac{1}{w}\cdot\frac{dw}{dt}=\lambda\left(1-\frac{w}{W}\right)$$

$$w=\frac{W}{1+ke^{-\lambda t}}$$

where

w ; weight of a plant　　　　λ ; growth factor

W ; maximum value of w　　　　t ; time

With some other assumptions, the following equation can be obtained.

$$\frac{1}{w}=A\rho+B \qquad \text{where } \rho\text{ ; number of plant per area}$$

This equation is called "reciprocal equation of competition-density effect". And from the relation

$$y=w\rho$$

the following equation is obtained.

$$\frac{1}{y}=A+\frac{B}{\rho}$$

This equation is called "reciprocal equation of yield-density effect". This theory was found by Kira et al. with herbaceous plants[1]. And in Japan this equation has been applied mainly to the even-aged pure softwood forest.

And the application of the reciprocal equation to some hardwood forests in France is discussed here and the stand density control diagram of beech in France is constructed.

The source of all data is in CNRF (Centre National de Recherches Forestieres).

Application of the reciprocal epuation to some hardwood forests

For the preliminary test, the author tried to apply this equation to the stand of Douglas fir (*Pseudotsuga Menziesii*) in France. The data was obtained in the experimental plot in Amance. The value of 1/V and 1/N were calculated. And

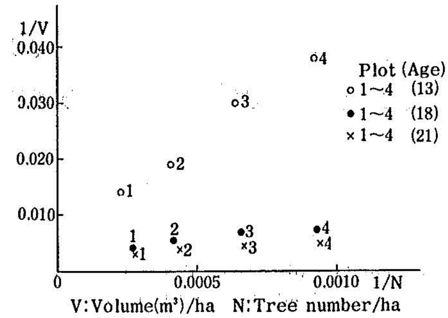

Fig. 1. The relation between 1/V and 1/N on the stand of Douglas fir in Amance (Data : CNRF)

Fig. 1 shows the relation between 1/V and 1/N at each growth stage. The reciprocal equation of yield-density effect can be adopted here quite exactly. The values of the correlation coefficient are more than 0.97 at all growth stages.

By the way, in France the main species of hardwood forest are oak and beech. Therefore these two species were used for this study. Fig. 2 shows the

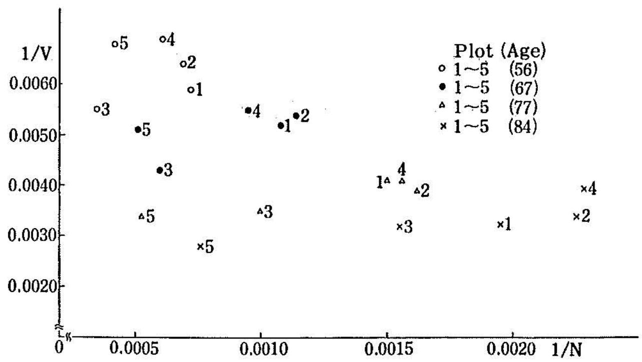

Fig. 2. The relation between 1/V and 1/N on the stand of Oak in Blois (Data : CNRF)

relation between 1/V and 1/N at each growth stage of Oak (*Quercus sessiliflora*) in Blois. The reciprocal equation fits to oak less than to Douglas fir. The values of the correlation coefficient are 0.15(56 years old), 0.60(67), 0.91(77) and 0.82(84).

The other data are found on oak in the State Forest of Champenoux. The data were obtained at two places. Fig. 3 shows the relation between 1/V and 1/N in each age class. The correlation coefficients are 0.46 (60 to 64 years old), 0.47 (81 to 83) and 0.83 (91).

The application of reciprocal equation for Beech (*Fagus silvatica*) is quite interesting. Because the beech in Europe seems to grow with only one storey. In

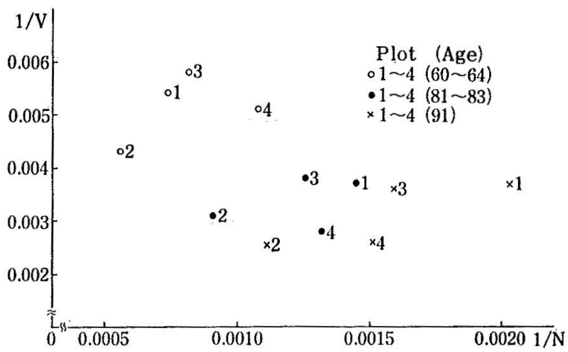

Fig. 3. The relation between 1/V and 1/N on the stand of Oak in Champenoux (Data : CNRF)

Japan, on the contrary, Beech (*Fagus crenata*) grows with many storeys. Therefore it is assumed that this equation fits to beech in France better than to that of Japan. Fig. 4 shows the relation between 1/V and 1/N at each growth stage of

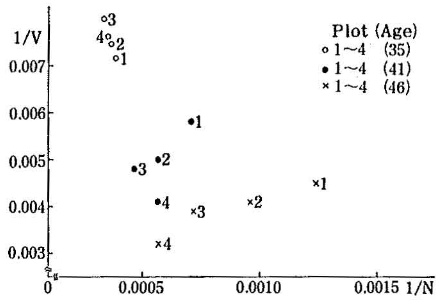

Fig. 4. The relation between 1/V and 1/N on the stand of Beech in Souilly (Data : CNRF)

beech in Souilly[4]. The correlation coefficients are 0.996 (35 years old), 0.65 (41), and 0.95 (46).

There is an old experimental stand of beech in the State Forest of "Eawy" in the north west of France. There, four experimental plots for thinning are found. From 1924 to up to date the thinning trial has been doing.

The treatments of thinning in each plot are as follows.

Plot 1 : heavy thinning (Danish thinning), rotation, 5 years
Plot 2 : light thinning, rotation, 10 years
Plot 3 : heavy thinning, rotation, 10 years
Plot 4 : light thinning (local thinning), rotation, 10 years

And it should be noted that the plot 4 abutting on two roads directly. Fig. 5 shows the relation between 1/V and 1/N at each growth stage before thinning.

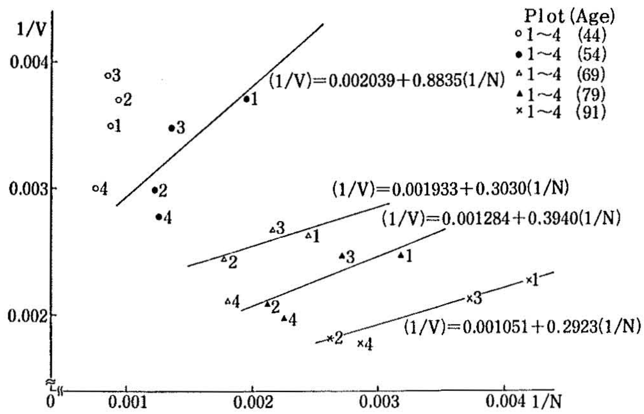

Fig. 5. The relation between 1/V and 1/N on the stand of Beech in Eawy (Data : CNRF)

For three plots (except plot 4) the reciprocal equation is adopted at four growth stage, namely, at 54, 69, 79 and 91 years old. For the calculation of constants of the reciprocal equation of yield-density following formulae were used.

$$A=\frac{\sum y \sum w^2-\sum w \sum y \cdot w}{\sum y^2 \sum w^2-(\sum y \cdot w)^2}, \qquad B=\frac{\sum y^2 \sum w-\sum y \sum y \cdot w}{\sum y^2 \sum w^2-(\sum y \cdot w)^2}$$

where

$$w=\frac{y}{\rho}$$

The results are as follows.

54 years old $\quad \frac{1}{V}=0.002039+\frac{0.883467}{N}$

69 years old $\quad \frac{1}{V}=0.001933+\frac{0.302969}{N}$

79 years old $\quad \frac{1}{V}=0.001284+\frac{0.394041}{N}$

91 years old $\quad \frac{1}{V}=0.001051+\frac{0.292252}{N}$

The correlation coefficients calculated are 0.85 (54 years old), 0.83 (69), 0.90 (79) and 0.99997 (91). The reciprocal equation fits quite well especially to 91 years old.

Construction of the stand density control diagram of beech

The volume calculated by using the reciprocal equation at each growth stage is shown in Tab. 1. And the dominant heights at each growth stage are as follows. 25.1 m (54 years old), 29.1 m (69), 30.2 m (79) and 31.5 m (91).

The author constructed the stand density control diagram of beech in Eawy (Fig. 6). Here the X and Y axises are scaled as logarithm. The theoretical founda-

Table 1. Calculated V at each growth stage of Beech in Eawy

N/ha	V m³/ha		
	54 years old	79	91
100	92.0	191.4	251.7
200	153.9	307.3	398.1
300	200.7	385.0	493.8
400	235.4	440.7	561.3
500	262.7	482.6	611.4
600	284.8	515.3	650.2
700	302.9	541.4	681.0
800	318.1	562.9	706.1
900	331.1	580.8	726.9
1,000	342.2	595.9	744.5
1,200	360.3	620.2	772.5
1,400	374.5	638.8	
1,600	385.9	653.5	
1,800	395.3		
2,000	403.1		
2,200	409.8		

tion of this diagram is as follows. Ando[1)] indicated that "When the growth factors other than the stand density are equally supplied after a certain period of growth, the relations between average stem volume (v) or stem volum per ha (V) and stand density (ρ) are given by the following equations:

$$\frac{1}{v}=A_1\rho+B_1$$

$$\frac{1}{V}=A_1+\frac{B_1}{\rho}$$

where A_1 and B_1 are constants determined by growth stage". And we have known about the full-density curve, which shows the upper limit of stand density. They are shown in the following equations

$$v=k\rho^{-a}$$

$$V=k\rho^{(1-a)}$$

Ando[1)] also stated as follows. "In 1962, the author showed that the full-density curve was given from the relations of the equation

$$\frac{1}{v}=A_1\rho+B_1$$

$$A_1=a_1H^{-b_1}$$

$$B_1=a_1'H^{-b_1'}$$

by means of using $v \cdot B_1$ as the competition index (Rc) and Rf as the minimum competition index in the present stand.

That is

$$V_{Rf}=k_2'\rho^{-k_1}$$

$$k_1=b_1'/(b_1-b_1')$$

$$k_2'=\frac{R_f}{a_1}\left\{\frac{R_f}{1-R_f}\cdot\frac{a_1}{a_1'}\right\}^{\frac{b_1'}{b_1-b_1'}}$$

By these equations, the full-density curve could be related to the reciprocal equation mathematically".

"Furthermore, according to the yield index (Ry) which was shown by the ratio of a certain stem volume on the same equivalent height curve to the maximum stem volume per ha which was given by the intersecting point of full-density curve and a certain equivalent height curve, the equivalent yield index curves are given by the following equations:

$$V_{Ry}=k_3\rho_{Ry}^{k_1'}$$

$$k_1'=b_1/(b_1-b_1')$$

$$k_3=\frac{(1-R_f)}{a_1}\left\{\frac{a_1}{a_1'}\cdot\frac{1-(1-R_f)R_y}{(1-R_f)R_y}\right\}^{\frac{b_1}{b_1-b_1'}}\text{"}$$

In Fig 6, the curves obtained by the reciprocal equation at 54, 79 and 91

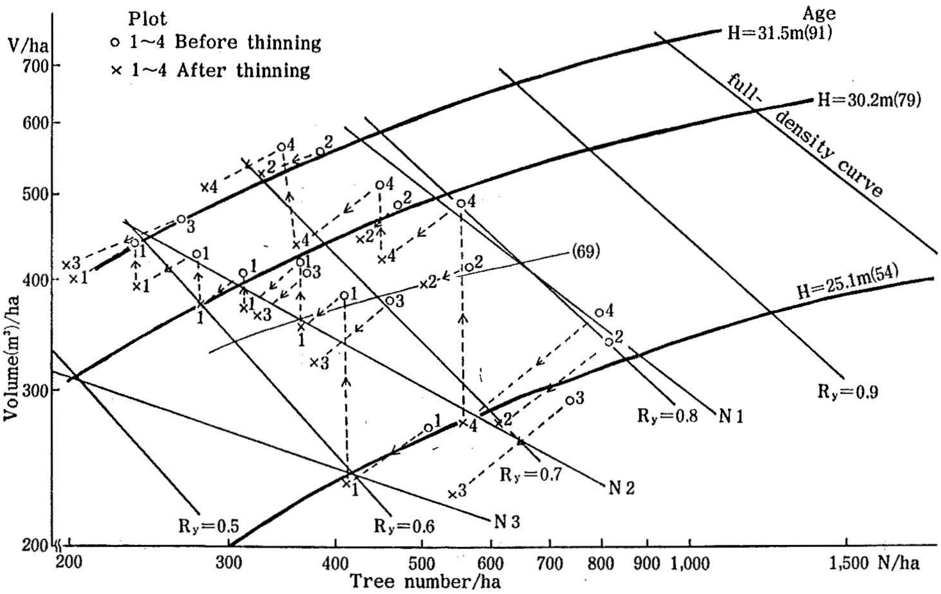

Fig. 6. Stand density control diagram of Beech forest in Eawy

years old are regarded as the dominant height curves. The lines of the full-density curve and Norm (N1 to N3) are drawn approximately by means of the Fig. 7 studied by H. Oswald[3]. And the lines of Ry=0.5 to Ry=0.9 are drawn depending on the full-density curve.

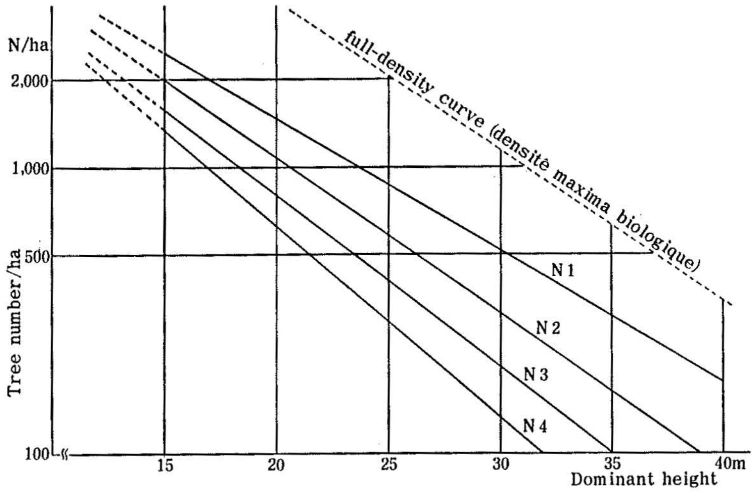

Fig. 7. Experimental norm for thinning in Beech forest (by. H. Oswald)

From Fig. 6 following points can be cleared.

1. This diagram shows the relations among dominant height, number of trees per ha and volume per ha.
2. The inclinations of the lines of the Norm (N 1 to N 3) differ one another. On the contrary, depending on the theory of Ando all the lines of yield index (Ry) run parallel.
3. In general, past thinning trial fits to the guide line of Norms. But the author propose a new thinning method which fits to the guide line of yield index (Ry).

Acknowledgement

I should like to express my heartfelt thanks to Prof. Dr. J. Pardé and Dr. H. Oswald for supporting my work in France and supplying data for this study.

Summary

In Japan the stand density control diagram which was constructed by Ando has been used generally. And the theoretical foundation of this diagram is the reciprocal equation, which is derived from the logistic curve.

The author tested the fitness of the reciprocal equation on some hardwood forest in France. For beech stand, the equation fits quite well. And the author constructed the stand density control diagram for a experimental stand of beech. And a new thinning method which fits the line of the yield index (Ry) in this diagram was proposed.

References

1) Ando, T. (1968). Ecological studies on the stand density control in even-aged pure stand, Bull. Jap. For. Exp. Sta. no. 210, pp. 153
2) Oswald, H. et Divoux, A. (1978). Dispositif experimental d'éclaircies du Hêtre "Camp Cussion" en F. D. d'EAWY (Seine-Maritime), INRA-CNRF, pp. 9
3) Oswald, H. (1980). Modèles de sylviculture et normes de densité, INRA-CNRF, pp. 6
4) Pardé, J., Oswald, H. et Tisserand, H. (1979). Le "Carré Latin" de Souilly—Dispositif expérimental d'éclaircie du Hêtre—, INRA-CNRF, pp. 9

フランスの広葉樹林の林分密度管理図作成に関する研究

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摘　　要

林分密度管理図は我国の多くの樹種に関して作成されており，その理論的基礎は植物の生長がロジスチック曲線に従うこと．それにともない密度と植物量との間に逆数式が成り立つことにある．ところでこの理論は生長が一様であることが前提となるため，我国では主として針葉樹の一斉林について応用されてきた．

ところでフランスの広葉樹林，とくにブナ林ではその生長の過程において階層分化を行なわず一様に生長することが観察されている．そこでまず逆数式をフランスのいくつかのブナ，ナラ林のデータに用い，その適合性をみた．その結果ブナ林ではこの式がきわめてよく適合することがわかった．そこでブナの間伐試験地のデータをもとにこの式を用いた林分密度管理図を作成した．さらにここで従来のフランスの間伐基準（Norme）とは別に最多密度曲線に基づく間伐の基準（Ry）による間伐方法を提示した．